

Effect of Eccentricity Fluctuations and Nonflow on Elliptic Flow Methods

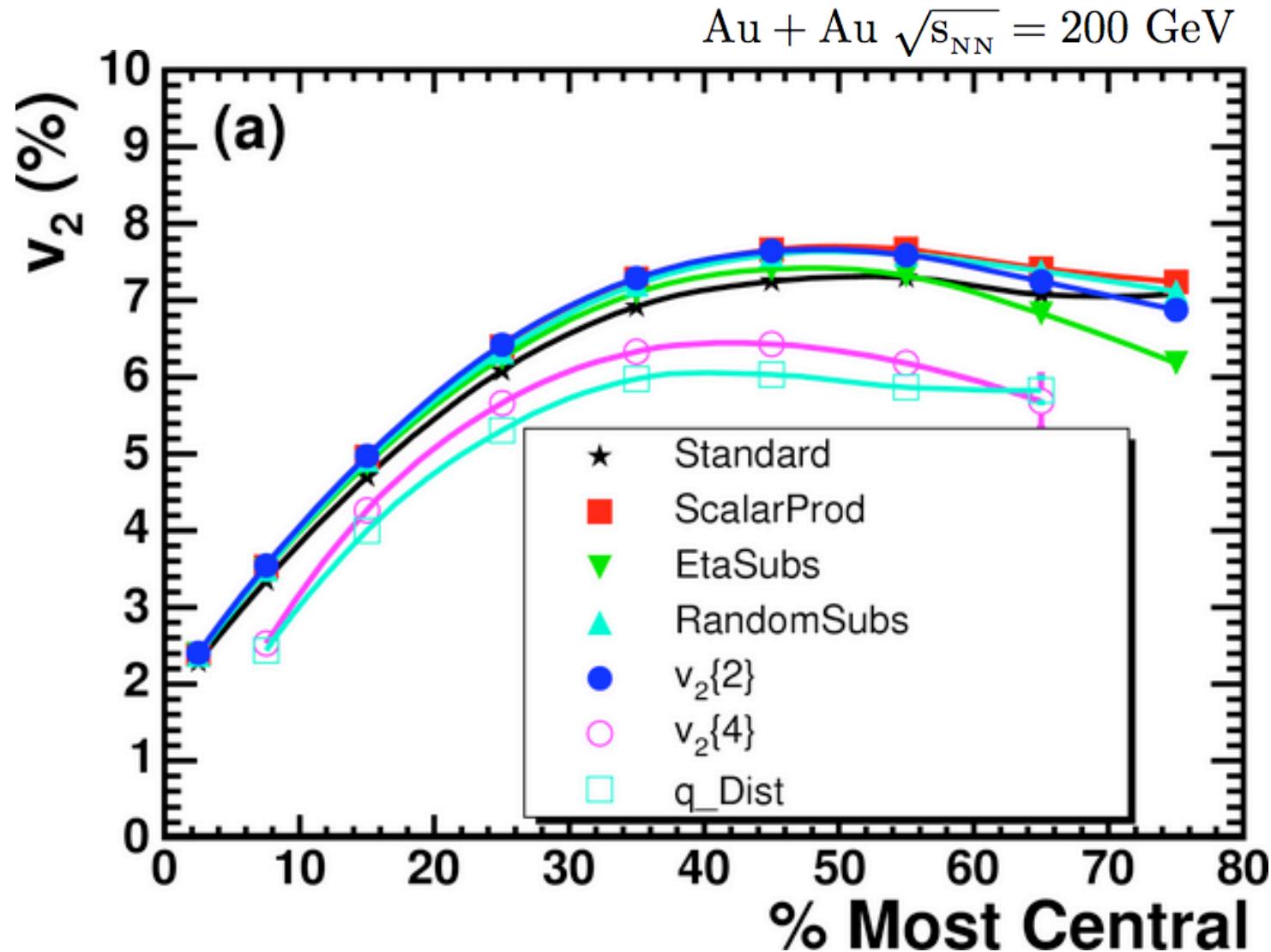
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TECHQM 15 Dec 08

Methods

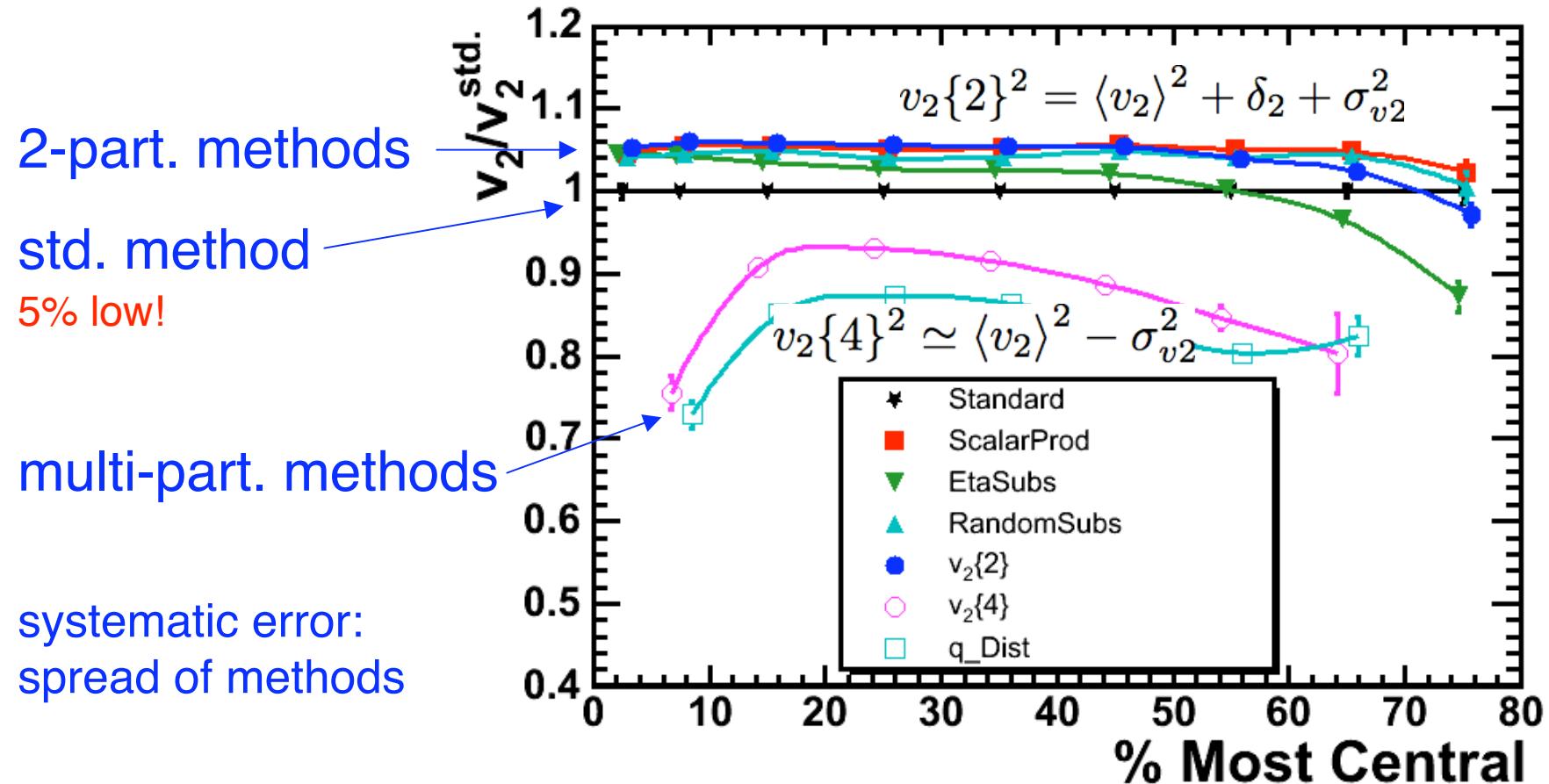
- “Two-particle”:
 - $v_2\{\text{subEP}\}$: each particle with the EP of the other subevent
 - $v_2\{\text{EP}\}$ “standard”: each particle with the EP of all the others
 - $v_2\{\text{SP}\}$: same, weighted with the length of the Q vector
 - $v_2\{2\}$: each particle with every other particle
- Many-particle:
 - $v_2\{4\}$: 4-particle - 2 * (2-particle)²
 - $v_2\{q\}$: distribution of the length of the Q vector
 - $v_2\{\text{LYZ}\}$: Lee-Yang Zeros multi-particle correlation

Integrated v_2



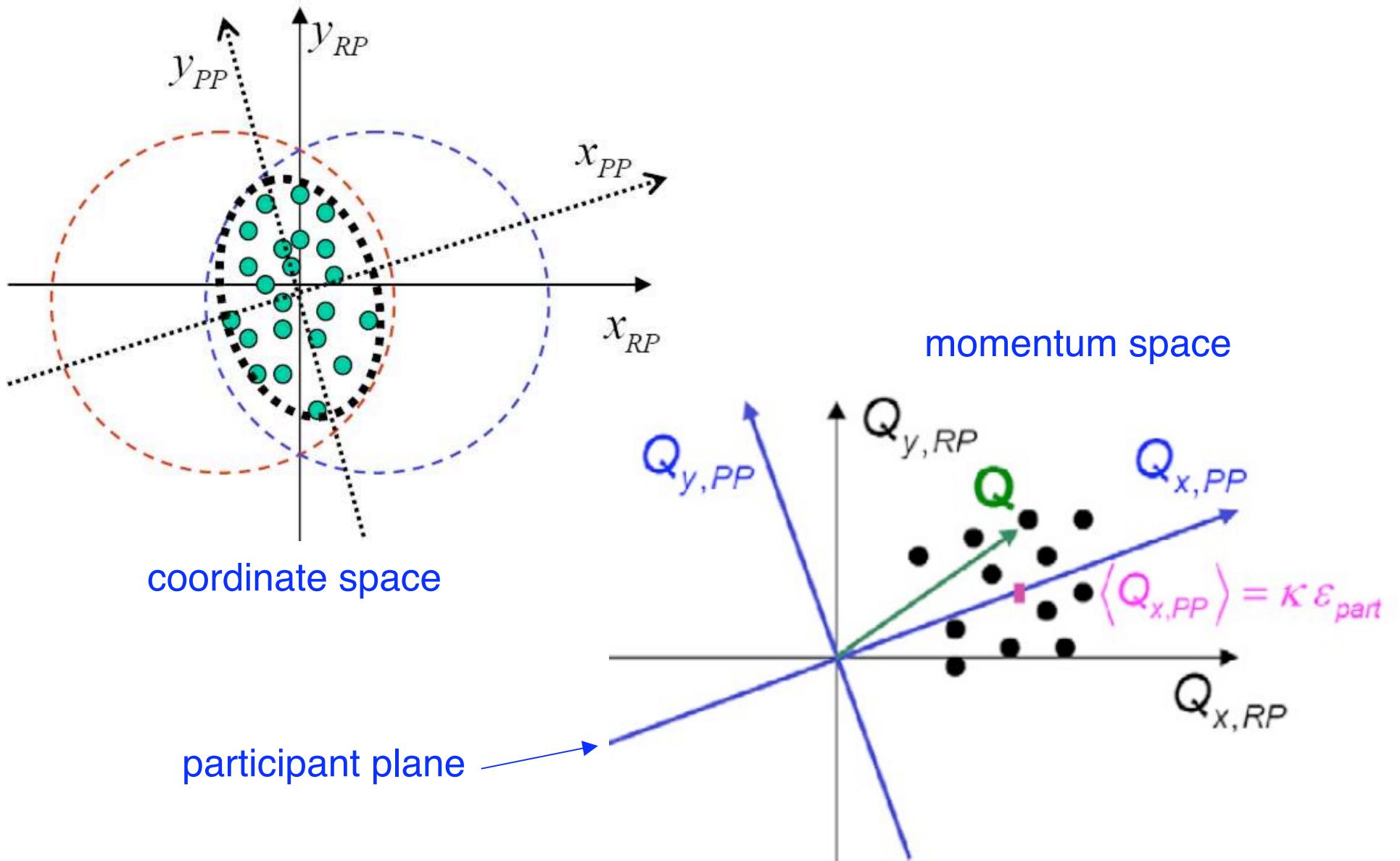
Methods Comparison (2005)

To expand scale, plot ratio to Standard Method:



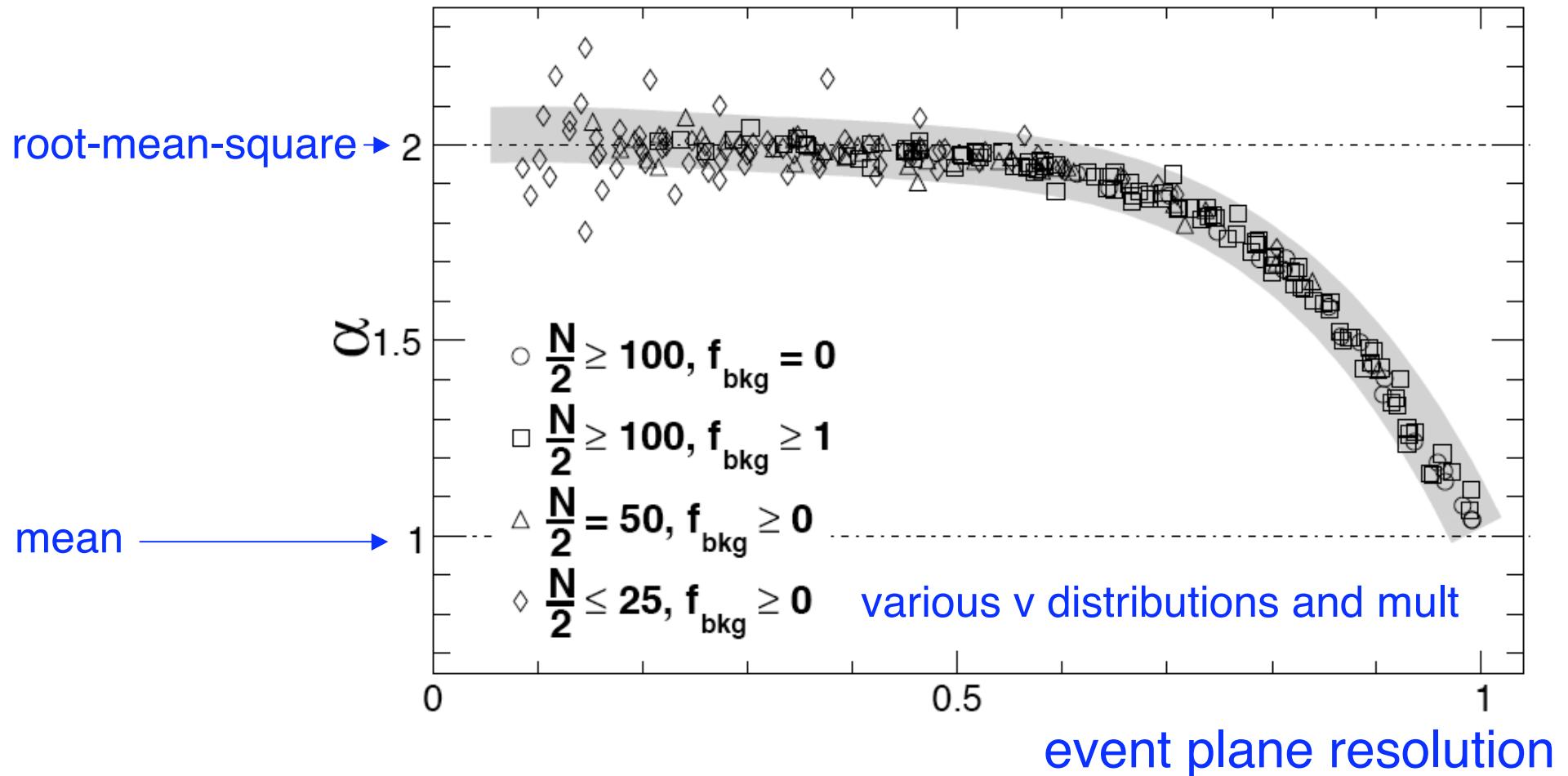
Because of nonflow and fluctuations the true v_2 lies between the lower band and the mean of the two bands.

Reaction, Participant, and Event Planes



PHOBOS Simulations

$$v_2\{\text{subEP}\} = \langle v_2^\alpha \rangle^{1/\alpha}$$



Include Fluctuations

$$v = \frac{v^{\text{obs}}}{\text{res}}$$

$$\langle \cos(\phi - \Psi_{\text{EP}}) \rangle = \langle \cos(\phi - \Psi_{\text{RP}}) * \cos(\Psi_{\text{EP}} - \Psi_{\text{RP}}) \rangle$$

$\langle v^{\text{obs}} \rangle$ v res

$$v\{\text{subEP}\} = \frac{\langle v \text{ res}(v) \rangle}{\sqrt{\langle \text{res}^2(v) \rangle}}$$

square-root of
subevent corr.

$= v$ in absence of fluctuations

The α Equation for Subevents

$$\alpha = 1 + \frac{\langle v \rangle \text{res}'}{\text{res}} \left[2 - \frac{\langle v \rangle \text{res}'}{\text{res}} \right]$$

$$\chi = v \sqrt{M}$$
 resolution parameter

$$x = \chi^2 / 2$$

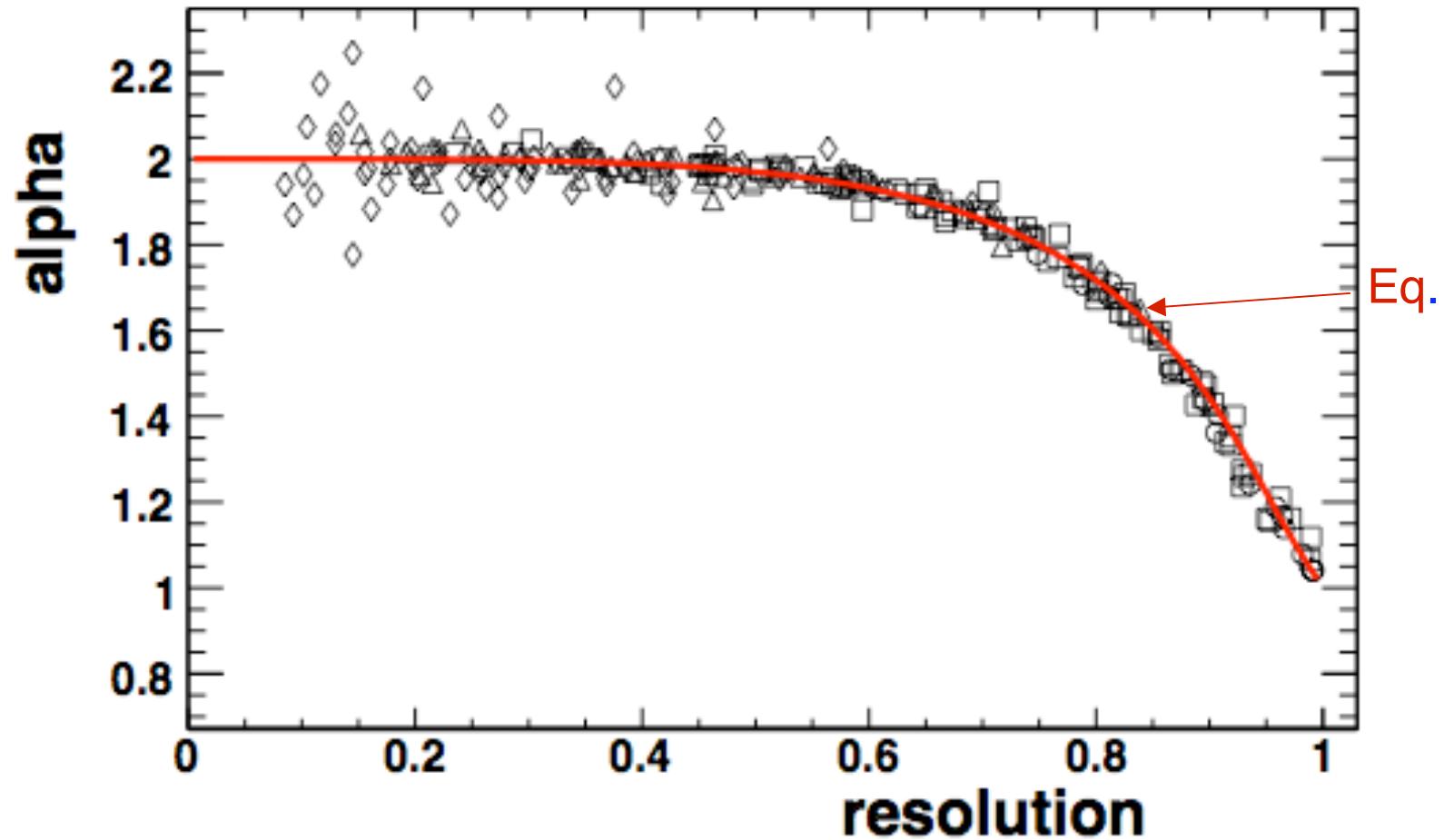
$$\text{res} = \sqrt{\pi}/2 \chi \exp(-x) (I_0(x) + I_1(x))$$

$$\alpha = 2 - \frac{4I_1(x)^2}{[I_0(x) + I_1(x)]^2}$$
 only function of χ

$I_{0,1}$ are modified Bessel functions

for full events, it is more complicated

PHOBOS+ with Equation



Fluctuations!

Experiment - Theory

- **Experiment**
 - correct for event plane resolution
 - nonflow problem
 - fluctuations: measure mean of some power of the distribution
- **Theory**
 - usually a value in the reaction plane
- **We must communicate**

Analytic Correction for Fluctuations

$$I_{0,1} = I_{0,1}(\chi^2/2) \quad i_{0,1} = I_{0,1}(\chi_s^2/2)$$

$$v_2\{2\}^2 = \langle v_2 \rangle^2 + \sigma_{v2}^2$$

$$v_2\{4\}^2 \simeq \langle v_2 \rangle^2 - \sigma_{v2}^2$$

$$v\{\text{subEP}\} = \langle v \rangle + \left(1 - \frac{4I_1^2}{(I_0 + I_1)^2} \right) \frac{\sigma_v^2}{2\langle v \rangle}$$

$$v\{\text{EP}\} = \langle v \rangle + \left(1 - \frac{I_0 - I_1}{I_0 + I_1} \left(2\chi^2 - 2\chi_s^2 + \frac{4i_1^2}{i_0^2 - i_1^2} \right) \right) \frac{\sigma_v^2}{2\langle v \rangle}$$

method similar to momentum conservation correction:

N. Borghini, P.M. Dinh, J.-Y. Ollitrault, A.M. Poskanzer, and S.A. Voloshin,
PRC **66**, 014901 (2002)

Analytic Correction for Nonflow

$$\langle \cos(\phi_1 - \phi_2) \rangle = \langle v \rangle^2 + \delta \quad \text{nonflow}$$

$$v_2\{2\}^2 = \langle v_2 \rangle^2 + \delta_2$$

$$v_2\{4\} = \langle v_2 \rangle$$

$$v\{\text{subEP}\} = \langle v \rangle + \left(1 - \frac{2I_1^2}{(I_0 + I_1)^2}\right) \frac{\delta}{2\langle v \rangle}$$

$$v\{\text{EP}\} = \langle v \rangle + \left(1 - \frac{I_0 - I_1}{I_0 + I_1} \left(\chi^2 - \chi_s^2 + \frac{2i_1^2}{(i_0^2 - i_1^2)} \right)\right) \frac{\delta}{2\langle v \rangle}$$

$$\langle v \rangle = v$$

Differences of Measured v_2 Values

$$\begin{aligned} v\{2\}^2 - v\{4\}^2 &= \delta + 2\sigma_v^2 \\ v\{2\}^2 - v\{\text{EP}\}^2 &= \frac{(I_0 - I_1)}{(I_0 + I_1)} \left(\chi^2 - \chi_s^2 + \frac{2i_1^2}{(i_0^2 - i_1^2)} \right) (\delta + 2\sigma_v^2) \\ v\{2\}^2 - v\{\text{subEP}\}^2 &= \frac{2I_1^2}{(I_0 + I_1)^2} (\delta + 2\sigma_v^2) \\ v\{\text{subEP}\}^2 - v\{\text{EP}\}^2 &= \frac{(I_0 - I_1)}{(I_0 + I_1)} \left(\chi^2 - \chi_s^2 + \frac{2i_1^2}{(i_0^2 - i_1^2)} - \frac{2I_1^2}{(I_0^2 - I_1^2)} \right) (\delta + 2\sigma_v^2) \end{aligned}$$

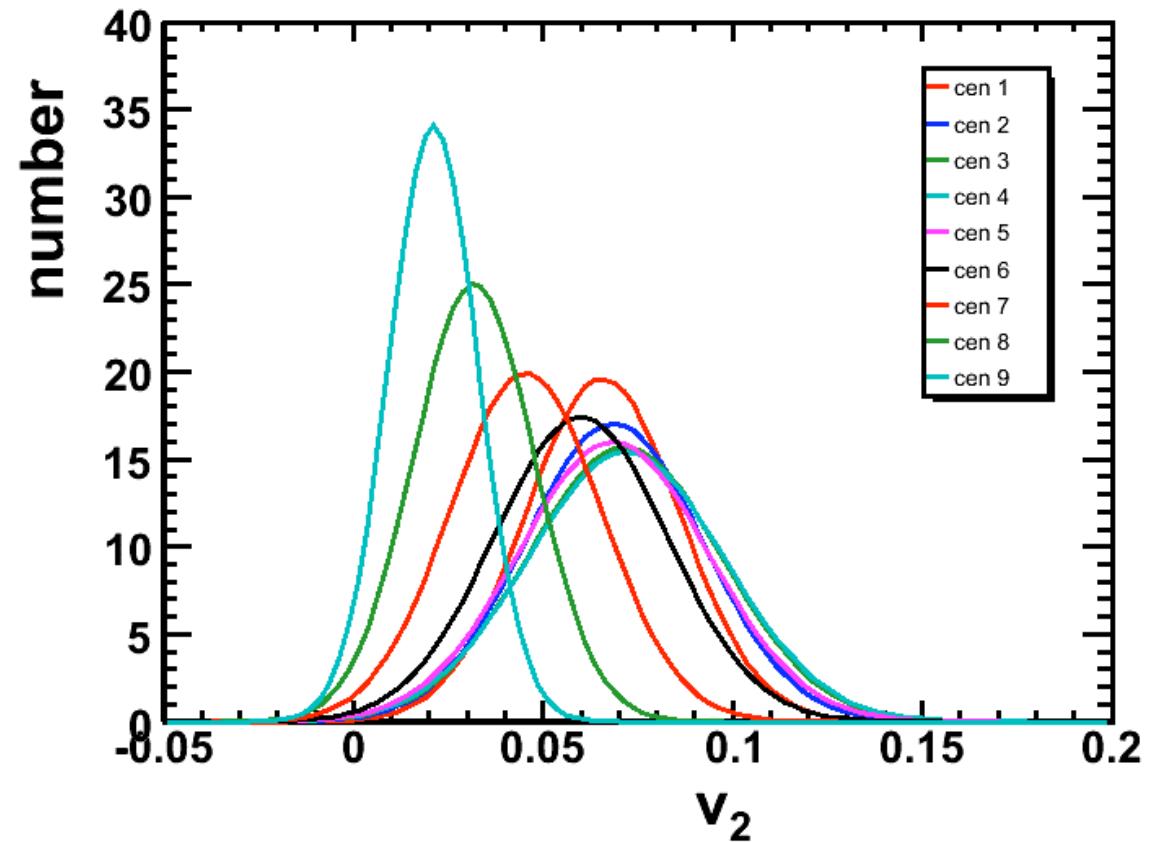
All differences proportional to $\sigma_{\text{tot}}^2 = \delta_2 + 2\sigma_{v2}^2$

Without additional assumptions
can not separate nonflow and fluctuations

Gaussian Fluctuations

Assume Gaussian with same percent width as $\varepsilon_{\text{part}}$: $\sigma_{v2} = \frac{\sigma_\varepsilon}{\langle \varepsilon \rangle} \langle v_2 \rangle$
 σ_ε is from standard deviation of nucleon MC Glauber of $\varepsilon_{\text{part}}$

along the
participant plane axis



Application to Data

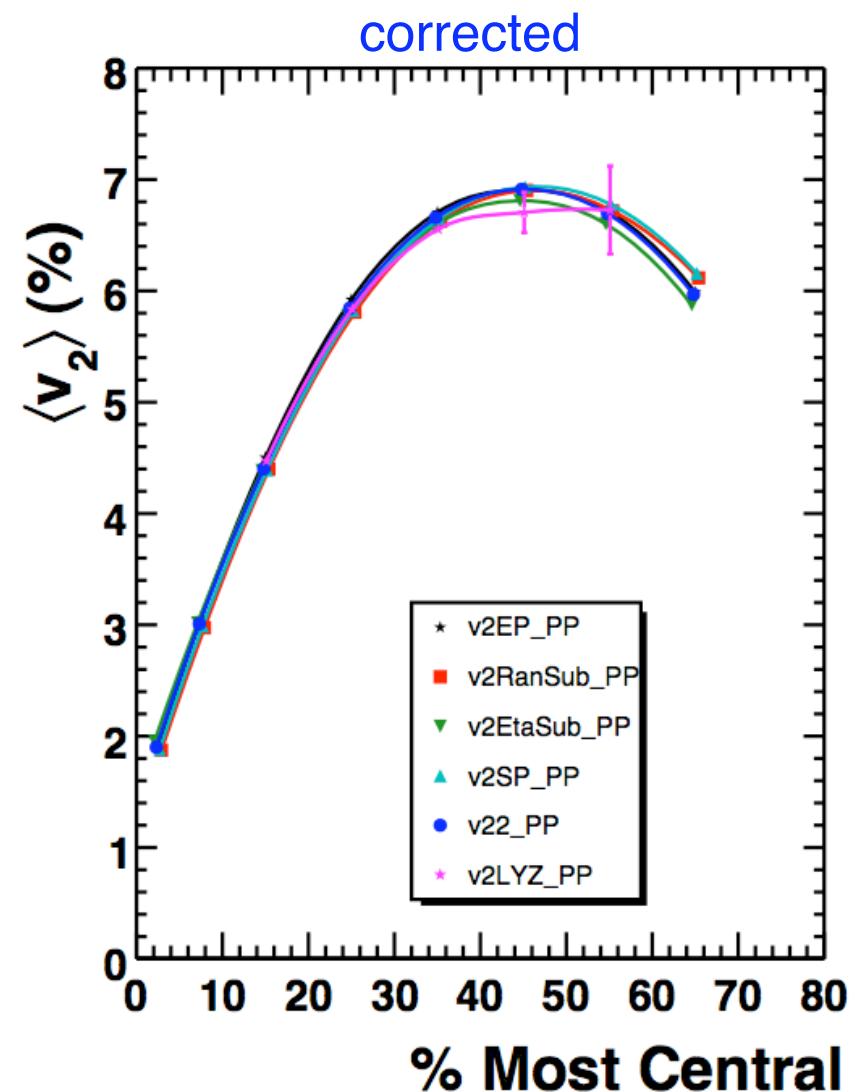
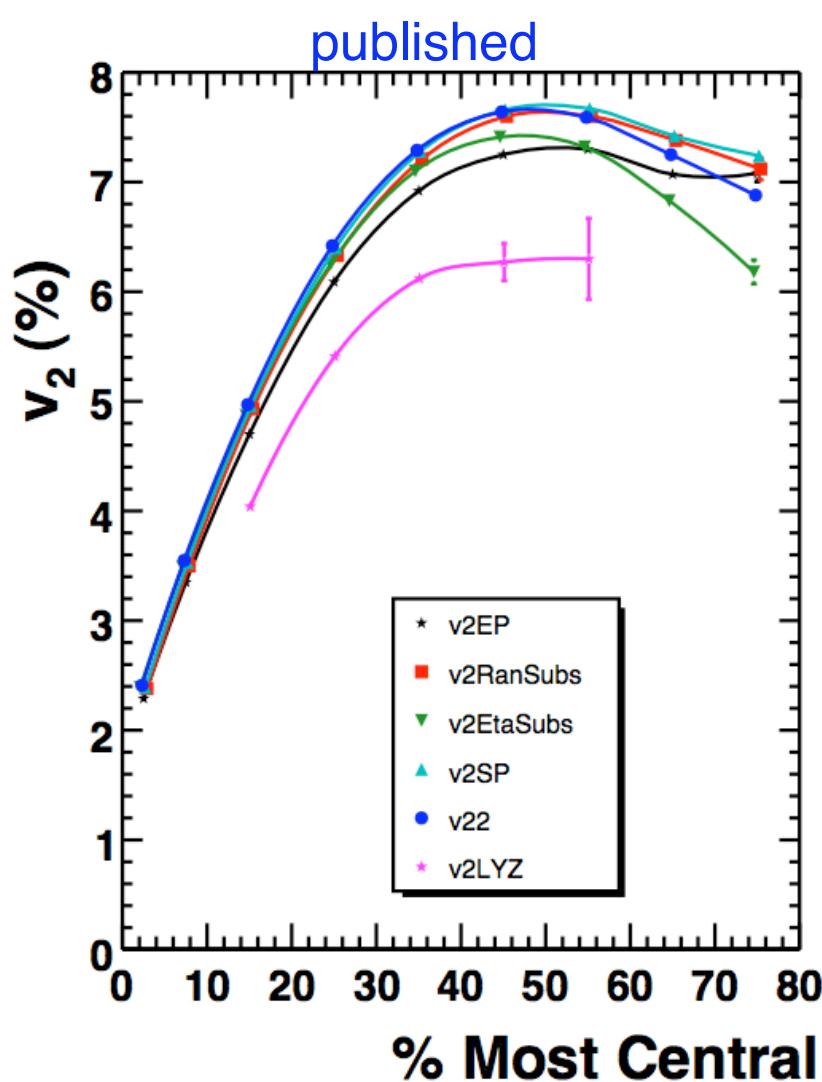
- Assumptions

$$\sigma_{v2} = \frac{\sigma_\varepsilon}{\langle \varepsilon \rangle} \langle v_2 \rangle \quad \text{MC Glauber } \varepsilon \text{ participant}$$

$$\delta_2 = 2 \delta_{pp}/N_{\text{part}} \quad \delta_{pp} = 0.0145$$

$$\delta_{\text{etaSub}} = 0.65 \delta_2 \quad \text{less nonflow}$$

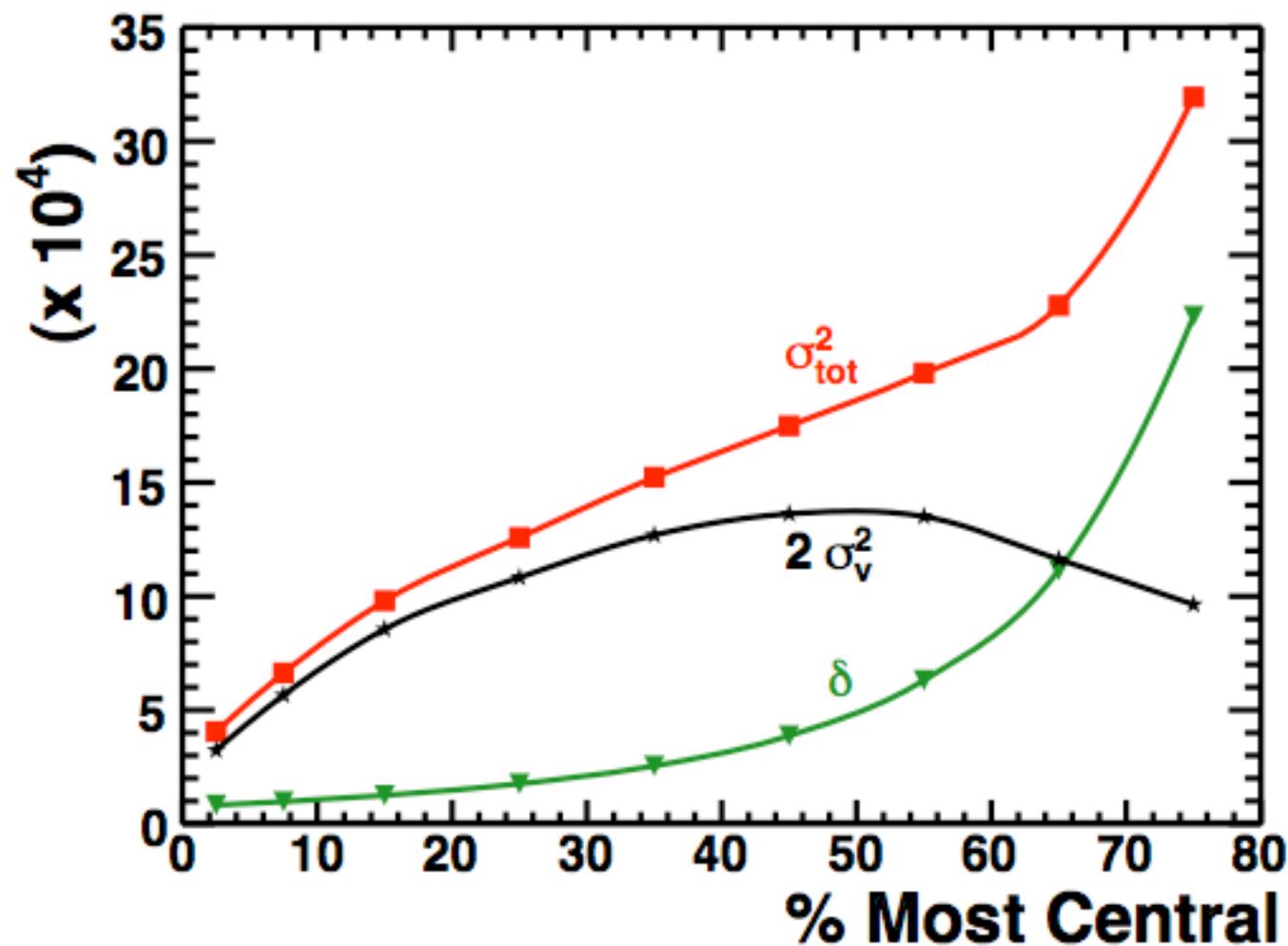
Data Corrected to $\langle v_2 \rangle$



agreement for mean v_2 in participant plane 16

Nonflow and Fluctuations

with my assumptions and parameters:

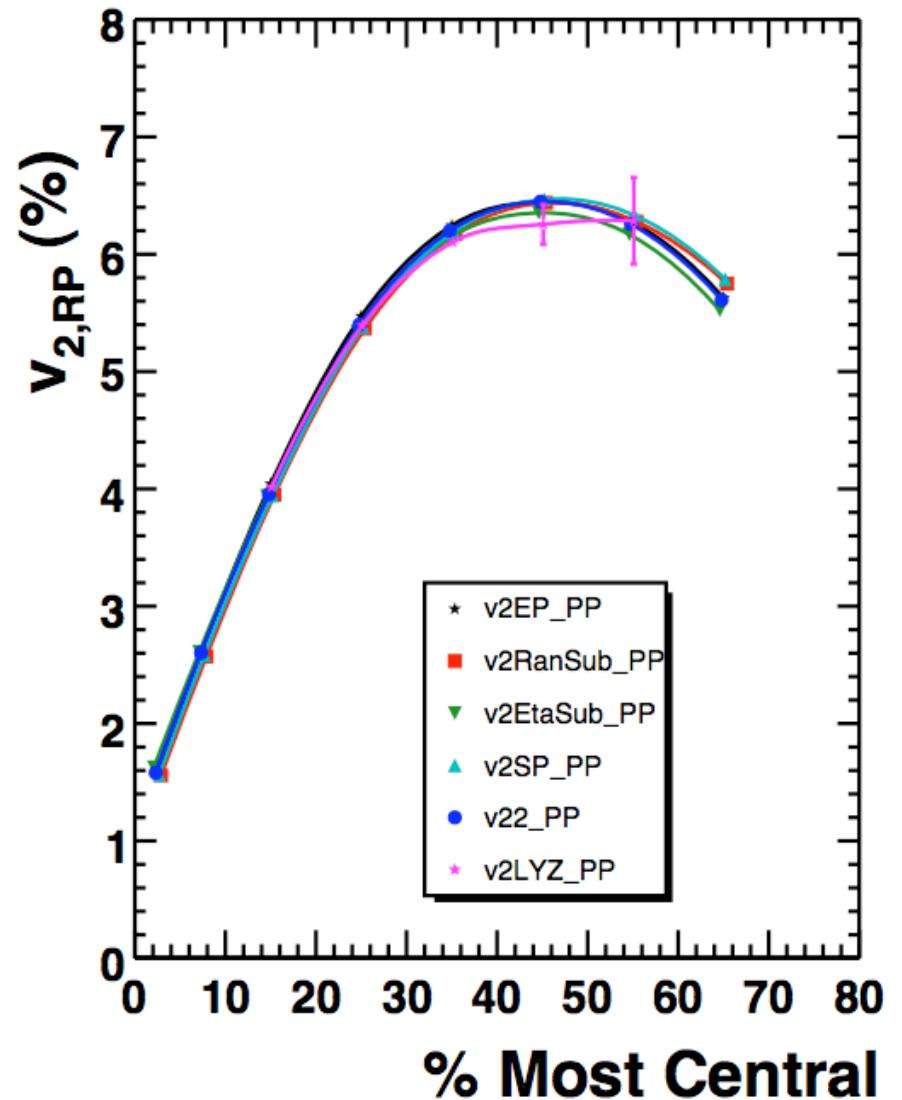


v_2 in the Reaction Plane

in Gaussian fluctuation approximation

$$v_{2,\text{PP}}^2 \simeq v_{2,\text{RP}}^2 + \sigma_{v2}^2$$

Finally, a v_2 for theorists



Conclusions

- Understanding of effects of fluctuations and nonflow
- All corrections proportional to σ_{tot}^2
- Can not separate δ and σ_v without assumptions
- $\langle v_2 \rangle$ consistent from different measurements with
 - $\varepsilon_{\text{part}}$ fluctuations from MC Glauber
 - $\delta = 2 \delta_{\text{pp}} / N_{\text{part}}$ and etaSub less
- Important for measurements with respect to event plane:
 - HBT, R_{AA} , conical emission, etc.
- Better multi-particle correlations are needed

Extra

Project Chronology

- I asked Constantin Loizides of PHOBOS
- He said Ollitrault understood
- I asked Jean-Yves
- Sergei rederived it elegantly
- I asked, does it explain why $v_2\{\text{EP}\}$ is 5% low?
- I did numeric integrations
- Jean-Yves derived equations
- I applied them to published STAR data
- Sergei will do simulations to test equations

Numeric Correction for Fluctuations

If one knows $\langle v \rangle$ and σ :

$$v\{\text{subEP}\} = \frac{\langle v \mathcal{R} \rangle}{\sqrt{\langle \mathcal{R}^2 \rangle}} \quad \chi = v \sqrt{M}$$

$$v\{\text{subEP}\} = \frac{\langle v \mathcal{R}(v \sqrt{M/2}) \rangle}{\sqrt{\langle [\mathcal{R}(v \sqrt{M/2})]^2 \rangle}}$$

$$v\{\text{EP}\} = \frac{\langle v \mathcal{R}(v \sqrt{M}) \rangle}{\mathcal{R} \left[C \left(\sqrt{\langle [\mathcal{R}(v \sqrt{M/2})]^2 \rangle} \right) \sqrt{2} \right]}$$

$$v\{2, \text{SP}\} = \sqrt{\langle v^2 \rangle} \quad \langle v \rangle = v\{2, \text{SP}\} / \sqrt{1 + \sigma_\varepsilon^2 / \langle \varepsilon \rangle^2}$$

Assume $\sigma_{v2} = \frac{\sigma_\varepsilon}{\langle \varepsilon \rangle} \langle v_2 \rangle$

and solve for $\langle v_2 \rangle$

not restricted to a Gaussian